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The self-similar problem of the diffusion mixing of vapour–gas–condensate systems☆

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ABSTRACT

Self-similar solutions of the problem of the diffusion mixing of vapour–gas–condensate mixtures, which is a generalization of the Stefan problem, are constructed. It is established on the basis of the solution of the problem concerning the mixing a vapour with a gas that, depending on the initial temperatures, mixing can occur with the formation of an intermediate vapour–gas–condensate layer. A chart of the possible structures of the mixing zones of a vapour–gas–condensate system with a vapour-gas mixture is obtained.

The mixing of air flows of different vapour contents and temperatures is frequently accompanied by phase transitions with the formation or disappearance of condensate in the form of fine droplets. These phenomena are fundamental in the formation and dispersion of fogs and clouds.[1] The visual configurations of surges into the atmosphere from exhaust pipes and discharge ejections are determined by condensation processes. Furthermore, the phenomenon of condensation accompanying the mixing of gaseous systems is of considerable importance in the preparation of nanomaterials.

The purely diffusion problem of the mixing of vapour–gas–condensate mixtures is considered below in a one-dimensional self-similar formulation for different values of the temperature at the initial instant.

1. Basic equations

In describing the mixing processes of vapour-gas condensed systems, we will make the following assumptions. We shall assume that a mixture consists of three components: a gas, which does not participate in phase transitions, and a vapour and a condensate, between which phase transitions can occur under equilibrium conditions. Suppose ρ_g, ρ_v and ρ_l are the partial (mean) densities of the gas, the vapour and the condensate. The volume concentration of the condensate in the form of drops ($\alpha_l = \rho_l/\rho_l^0$, where ρ_l^0 is the true density of the condensate) is low ($\alpha_l \ll 1$) and it therefore does not have an effect on the mixing of the gas and the vapour which occurs under diffusion conditions according to the equation

$$\frac{\partial \rho_g}{\partial t} = D\frac{\partial^2 \rho_g}{\partial x^2} \tag{1.1}$$

where D the diffusion coefficient. We also assume that the drops are stationary $v_l = 0$.

We will write the heat conduction equation taking into account phase transitions as

$$\rho c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2} + l\frac{\partial \rho_l}{\partial t}; \quad \rho c = \rho_g c_g + \rho_v c_v + \rho_l c_l \tag{1.2}$$

where T is the temperature, ρc is the heat capacity per unit volume of the mixture (c_g, c_v, c_l are the specific heat capacities of the components), λ is the thermal conductivity and l is the specific heat of the phase transitions. The second term on the right-hand side of Eq. (1.2)

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expresses a volume source ($\partial\rho_l/\partial t > 0$) or sink ($\partial\rho_l/\partial t > 0$) of heat on account of the latent heat of the phase transitions and, in zones where there is no condensate, it is equal to zero $\rho_l = 0$.

We shall also assume that the total pressure P, which consists of the partial pressures of the gas P_g and the vapour P_υ, is constant and, at the same time, the Mendeleyev-Clapeyron equation and Dalton's law are satisfied in the case of the gas and the vapour. We then have

$$P = P_g + P_\upsilon, \quad P_g = \rho_g RT/\mu_g, \quad P_\upsilon = \rho_\upsilon RT/\mu_\upsilon \tag{1.3}$$

where R is the universal gas constant, and μ_g and μ_υ are the molar masses of the gas and the vapour.

In the mixing zones, where there is condensate ($\rho_l > 0$), phase transitions occur under equilibrium conditions in accordance with the statements noted above (the vapour-gas mixture is at the dew point throughout the whole extent of this zone) and the partial pressure of the vapour P_υ is therefore equal to the pressure of the saturated vapour $P_s(T)$ corresponding to the current temperature $T(P_\upsilon = P_s(T))$. The expression[2]

$$P_s(T) = P_* \exp(-T_*/T) \tag{1.4}$$

is conventionally used where P_* and T_* are empirical parameters which are determined on the basis of tabulated data. It then follows from relations (1.3) that, in zones where ($\rho_l > 0$), the partial gas and vapour densities are uniquely defined in terms of the current temperature T in the form

$$\rho_\upsilon = \mu_\upsilon P_s(T)/(RT), \quad \rho_g = \mu_g(P - P_s(T))/(RT) \tag{1.5}$$

These expressions have a meaning when the condition $P_s(T) \le P$ is satisfied (otherwise ($P_s(T) > P$) and the value of the gas density ρ_g becomes negative). The value of the temperature $T = T_b$, at which the exact equality $P_s(T_b) = P$ is satisfied, corresponds to the boiling point at a pressure P.

Hence, the possible values of the partial vapour density at temperatures $T < T_b$ satisfy the condition

$$\rho_\upsilon \le \mu_\upsilon P_s(T)/(RT) \tag{1.6}$$

and, in the case of temperature above the boiling point ($T > T_b$), the condition

$$\rho_\upsilon \le \mu_\upsilon P/(RT) \tag{1.7}$$

The hatched region in the T, ρ_υ plane in Fig. 1 corresponds to the possible states of the vapour-gas condensed system at a pressure P, and the upper boundary of this region when $T \le T_b$ corresponds to the state of the system when there is condensate present ($\rho_l > 0$). The segment of the upper boundary when $T > T_b$ corresponds to a state of pure superheated vapour under a pressure P.

2. Mixing of vapour with gas

Suppose that, in the initial state, pure vapour with a density $\rho_{\upsilon o}$ at a temperature T_o is in the semi-infinite zone on the left of an imaginary partition ($-\infty < x \le 0$) and that pure gas at a temperature T_e is on the right ($0 < x < \infty$). Parameters, referring to the initial state in the left and right zones, will henceforth be labelled with additional o and e subscripts. At the instant $t = 0$, the partition is removed and diffusion mixing occurs when $t > 0$. We write the initial state, which has been mentioned, in the form of the following initial conditions

$$\begin{aligned} &T = T_o, \quad \rho_\upsilon = \rho_{\upsilon o} \ (\rho_{go} = 0) \text{ when } x < 0 \\ &T = T_e, \quad \rho_g = \rho_{ge} \ (\rho_{\upsilon e} = 0) \text{ when } x > 0 \end{aligned} \tag{2.1}$$

The points of the upper boundary of the hatched area when $T > T_b$ in Fig. 1 correspond to the first of these, and the points on the abscissa when $T \le T_b$ correspond to the second. In constructing the solutions of the equations presented in the preceding section, we shall assume that, in the general case, there is a certain intermediate zone between the boundaries $x = x^{(l)}$ and $x = x^{(r)}$ where condensate is present. We will consequently assume that $\rho_l > 0$ in the domain $x^{(l)} < x < x^{(r)}$. At the same time, the laws of motion of the left-hand boundary $x^{(l)}(t)$ and right-hand boundary $x^{(r)}(t)$ are unknown functions of time.

Note that the scheme adopted and the corresponding theoretical mixing model exclude phenomena such as the convective motion of the whole mixture and, also, wave processes associated, for example, with "thermal shock" from the treatment. This simplified approach can

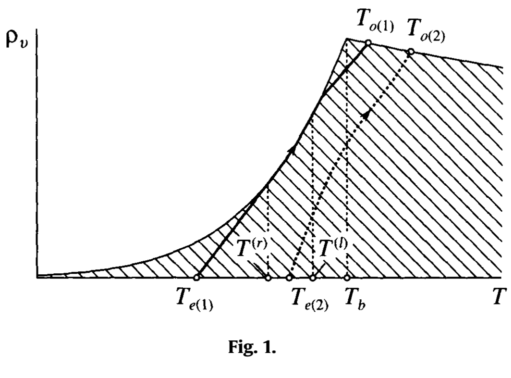

Fig. 1.

be used to analyse the possibility of the formation or dispersal of condensate accompanying the diffusion mixing of vapour-gas mixtures and, also, to estimate the characteristic dimensions of the mixing zones.

It follows from the conditions of continuity of the partial density of the gas and the diffusion flows of the gas on the two unknown boundaries ($x=x^{(l)}$ and $x=x^{(r)}$) that

$$\rho_g^- = \rho_g^+ = \rho_g^{(j)}, \quad D\left(\frac{\partial\rho_g}{\partial x}\right)^-_{x^{(j)}} = D\left(\frac{\partial\rho_g}{\partial x}\right)^+_{x^{(j)}}; \quad j = l, r \tag{2.2}$$

The − and + superscripts correspond to the values of quantities on the left and on the right of the boundaries.

We will assume that the partial density of the drops on the left-hand boundary ($x=x^{(l)}$) is equal to zero ($\rho_l^{(l)}=0$). The conditions for the continuity of the temperature and the heat fluxes on this boundary are then written in the form

$$T^- = T^+ = T^{(l)}, \quad -\lambda\left(\frac{\partial T}{\partial x}\right)^-_{x^{(l)}} + \lambda\left(\frac{\partial T}{\partial x}\right)^+_{x^{(l)}} = 0 \tag{2.3}$$

According to relations (1.5), in the region where there is condensate ($\rho_l > 0$), the partial vapour density is uniquely defined by the value of the current temperature. The gradients of the partial gas density and the temperature must then be related by the condition

$$\left(\frac{\partial\rho_g}{\partial x}\right)^+_{x^{(l)}} = \left(\frac{d\rho_g}{dT}\right)_{T^{(l)}}\left(\frac{\partial T}{\partial x}\right)^+_{x^{(l)}} \tag{2.4}$$

and, according to the second equality of (1.5),

$$\left(\frac{d\rho_g}{dT}\right)_{T^{(l)}} = -\frac{\mu_g}{RT^{(l)}}\left(\frac{P-P_s(T^{(l)})}{T^{(l)}} + \frac{dP_s(T^{(l)})}{dT^{(l)}}\right) \tag{2.5}$$

We will assume that the temperature is continuous on the right-hand boundary ($x=x^{(r)}$) and that the partial density of the drops can undergo a discontinuity. Suppose the partial density of the condensate on the left of the boundary is equal to $\rho_l^{(r)}$ (on the right, it is equal to zero). The temperature and heat balance continuity conditions on the boundary $x=x^{(r)}$ can then be written in the form

$$T^- = T^+ = T^{(r)}, \quad -\lambda\left(\frac{\partial T}{\partial x}\right)^-_{x^{(r)}} + \lambda\left(\frac{\partial T}{\partial x}\right)^+_{x^{(r)}} = -\dot{x}^{(r)}\rho_l^{(r)}l \tag{2.6}$$

Like to condition (2.4), we also have

$$\left(\frac{\partial\rho_g}{\partial x}\right)^-_{x^{(r)}} = \left(\frac{d\rho_g}{dT}\right)_{T^{(r)}}\left(\frac{\partial T}{\partial x}\right)^-_{x^{(r)}} \tag{2.7}$$

on the right-hand boundary.

This problem is a generalization of the Stefan problem[3] since the phase transitions here can not only occur in the frontal boundaries but also in spatial zones.

We shall seek a self-similar solution by introducing the self-similar dimensionless variable

$$\xi = \frac{x}{2\sqrt{Dt}} \tag{2.8}$$

From Eq. (1.1), we then obtain

$$-2\xi\frac{d\rho_g}{d\xi} = \frac{d^2\rho_g}{d\xi^2} \tag{2.9}$$

The heat flux equation (1.2) is written in the form

$$-2\xi\frac{dT}{d\xi} = \mathrm{Le}\frac{d^2T}{d\xi^2} - 2\xi\frac{l}{\rho c}\frac{d\rho_l}{d\xi}, \quad \mathrm{Le} = \frac{D\rho c}{\lambda} \tag{2.10}$$

The initial conditions (2.1) are transformed into the following boundary conditions for Eqs. (2.9) and (2.10)

$$\rho_g = 0, \quad T = T_0 \text{ when } \xi\to-\infty, \quad \rho_g = \rho_{ge}, \quad T = T_e \text{ when } \xi\to+\infty \tag{2.11}$$

We shall assume that the unknown self-similar coordinates of the intermediate boundaries

$$\xi^{(l)} = \frac{x^{(l)}}{2\sqrt{Dt}}, \quad \xi^{(r)} = \frac{x^{(r)}}{2\sqrt{Dt}} \tag{2.12}$$

are constant. The boundary conditions (2.2) are then written in the form

$$\rho_g^- = \rho_g^+ = \rho_g^{(j)}, \quad \left(\frac{d\rho_g}{d\xi}\right)^-_{\xi^{(j)}} = \left(\frac{d\rho}{d\xi}\right)^+_{\xi^{(j)}} \text{ when } \xi = \xi^{(j)}; \quad j = l, r \tag{2.13}$$

For simplicity, we will assume that $Le \approx 1$ (this condition always holds for gaseous mixtures). From the heat balance equations (2.3) and (2.6), we then obtain

$$-\left(\frac{dT}{d\xi}\right)^-_{\xi^{(l)}} + \left(\frac{dT}{d\xi}\right)^+_{\xi^{(l)}} = 0, \quad -\left(\frac{dT}{d\xi}\right)^-_{\xi^{(r)}} + \left(\frac{dT}{d\xi}\right)^+_{\xi^{(r)}} = -2\xi^{(r)}\frac{\rho_l^{(r)} l}{\rho c} \tag{2.14}$$

The solution of Eq. (2.9), which satisfies conditions (2.11) and (2.13) in the whole of the mixing zone, has the form

$$\rho_g(\xi) = \frac{\rho_{ge}}{\sqrt{\pi}} E(-\infty, \xi), \quad E(\alpha, \beta) = \int_\alpha^\beta \exp(-z^2)dz \tag{2.15}$$

Using this solution, on the basis of the second relation of (1.5) and taking account of equality (1.4), we obtain the solution for the temperature distribution in the intermediate zone, where there is condensate, in the implicit form

$$\rho_g(\xi) = \frac{\mu_g}{RT}(P - P_s(T)) \tag{2.16}$$

Substituting this relation into the heat flux equation (2.10), we obtain a first-order differential equation for determining the partial condensate density. If the contribution $\rho_l c_l$ of the heat capacity due to the condensate is neglected in the total heat capacity of the mixture ρc, we can write the solution of this equation for the partial density distribution of the condensate in the form

$$\rho_l = \frac{1}{2}\int_{\xi^{(l)}}^{\xi} \frac{\rho c}{l} \frac{d^2 T}{d\rho_g^2}\left(\frac{d\rho_g}{d\xi}\right)^2 \frac{d\xi}{\xi} \tag{2.17}$$

and, by relations (2.15) and (2.16)

$$\frac{d^2T}{d\rho_g^2} = \frac{1}{T}\left(\frac{RT^2/\mu_g}{P - P_s(T)(1 - T_*/T)}\right)^2 \left(2 + \frac{P_s(T)(2 - T_*/T)T_*/T}{P - P_s(T)(1 - T_*/T)}\right)$$

$$\frac{d\rho_g}{d\xi} = \frac{\rho_{go}}{\sqrt{\pi}}\exp(-\xi^2)$$

The partial vapour density in this zone can be determined from the first expression of (1.5) using the known temperature distribution.

The temperature distributions in the zones $\xi < \xi^{(l)}$ and $\xi > \xi^{(r)}$, where there is no condensate, have the form

$$T = T_o + (T^{(l)} - T_o)\frac{E(-\infty, \xi)}{E(-\infty, \xi^{(l)})}, \quad T = T_e + (T^{(r)} - T_e)\frac{E(\xi, \infty)}{E(\xi^{(r)}, \infty)} \tag{2.18}$$

Using these temperature distributions, the vapour density in zones where there is only a vapour-gas mixture ($\rho_l = 0$) can be determined from expression (1.3):

$$\rho_\upsilon(\xi) = \mu_\upsilon\left(\frac{P}{RT(\xi)} - \frac{\rho_g(\xi)}{\mu_g}\right) \tag{2.19}$$

Note that the values of the self-similar coordinates of the intermediate boundaries $\xi^{(l)}$ and $\xi^{(r)}$ as well as the values of the temperatures on their boundaries, which are present in the solutions (2.17), still remain unknown.

From condition (2.4), taking account of equality (2.5) and, also, using the solutions (2.15) and (2.17), we obtain the following transcendental equation for determining the value of the temperature on the left boundary

$$\frac{T_o}{T^{(l)}} - 1 = \frac{P - P_s(T^{(l)})}{P - P_s(T^{(l)})(1 - T_*/T^{(l)})} \tag{2.20}$$

where the dew point is reached. It is noteworthy that the value of the temperature $T^{(l)}$ depends solely on the initial temperature of the vapour T_o and the pressure P. Using the value of $T^{(l)}$ found from the solution of Eq. (2.20) and substituting solution (2.15) into the left-hand side of expression (1.5) we obtain an equation for determining the self-similar coordinate $\xi^{(l)}$ of the left boundary in the form

$$\frac{\rho_{ge}}{\sqrt{\pi}}E(-\infty, \xi^{(l)}) = \frac{\mu_g}{RT^{(l)}}(P - P_s(T^{(l)})) \tag{2.21}$$

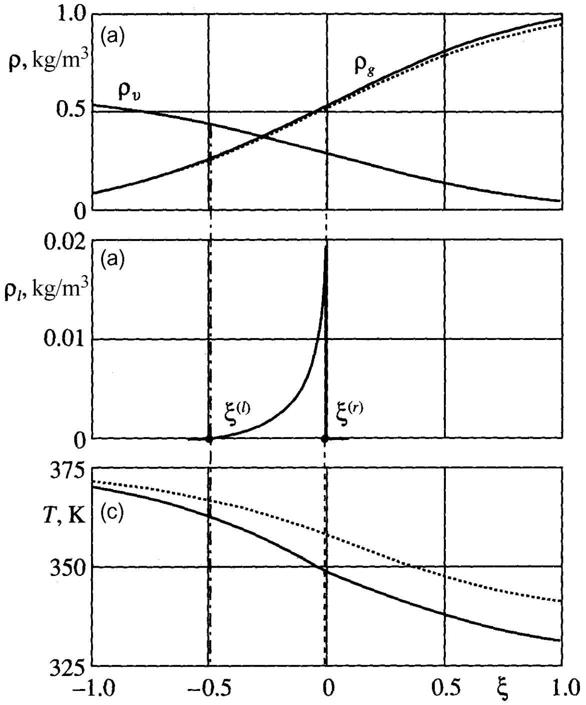

Fig. 2.

If the vapour is at saturation point in the initial state, its initial temperature satisfies the condition

$$P = P_s(T_o) \tag{2.22}$$

Then, as follows from the equality (2.20), the temperature $T^{(l)}$ will be equal to the initial temperature of the vapour T_o. In this case, we obtain from Eq. (2.15) that the left boundary has receded to infinity ($\xi^{(l)} \to -\infty$).

When account is taken of solutions (2.15) and (2.16), the boundary condition (2.14) can be reduced to the form

$$\left(\frac{dT}{d\rho_g}\right)_{T^{(r)}}\frac{\rho_{ge}}{\sqrt{\pi}}+\frac{T^{(r)}-T_{ge}}{E(\xi^{(r)},\infty)} = 2\xi^{(r)}\frac{\rho_l^{(r)}l}{\rho c}\exp((\xi^{(r)})^2) \tag{2.23}$$

where

$$\left(\frac{dT}{d\rho_g}\right)_{T^{(r)}} = -\frac{RT^{(r)2}}{\mu_g}\left(P-P_s(T^{(r)})\left(1-\frac{T_*}{T^{(r)}}\right)\right)^{-1}$$

From the solution (2.17) when $\xi = \xi^{(r)}$, we have

$$\rho_l^{(r)} = \frac{1}{2}\int_{\xi^{(l)}}^{\xi^{(r)}}\frac{\rho c}{l}\frac{d^2T}{d\rho_g^2}\left(\frac{d\rho_g}{d\xi}\right)^2\frac{d\xi}{\xi} \tag{2.24}$$

Equations (2.23) and (2.24) enable us to determine two unknown quantities: the self-similar coordinate of the right boundary $\xi^{(r)}$ and the partial condensate density $\rho_l^{(r)}$ on this boundary.

As an example, we will consider the mixing of dry water vapour ($T_o > T_b$) and dry air ($\rho_{ve} = 0$). The phase trajectories for the two modes of mixing are shown in Fig. 1. In the case of the mode which is determined by the initial temperatures $T_{o(1)}$ and $T_{e(1)}$, the mixing process occurs with the formation of an intermediate vapour–gas–condensate zone. The segment of the phase trajectory corresponding to this zone is located on the saturation line. When the initial temperatures $T_{o(2)}$ and $T_{e(2)}$ are sufficiently high, the phase trajectory does not fall on this line. In this case, an intermediate zone with condensate is not formed.

The results of calculations, which illustrate the structure of the mixing zone for the two modes, are shown in Fig. 2 (by the solid and dotted curves respectively). The partial vapour and air densities for these two modes differ insignificantly (Fig. 2, *a*). In the mode with condensation, only a relatively small amount of condensate is formed ($\rho_l \ll \rho_v, \rho_g$, see Fig. 2, *b*) and the maximum value of the density of the drops in the mixture reaches $\rho_l \approx 0.02\,\text{kg/m}^3$. This corresponds to the initial hypothesis that the volume fraction of the drops in the mixture is negligibly small (in the case in question $\alpha_l = \rho_l/\rho_l^0 \approx 2\times10^{-5}$). When there is condensation, the temperature distribution curve (Fig. 2, *c*) on the right boundary of the intermediate layer has a small kink because of the consumption of heat in evaporation on the boundary $\xi^{(r)}$ in accordance with boundary condition (2.23).

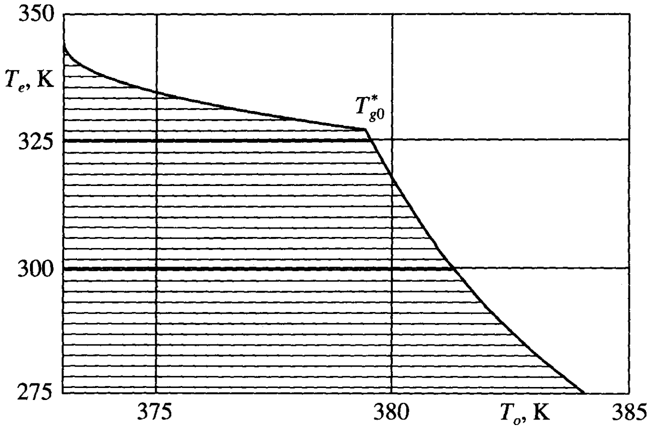

Fig. 3.

The mixing of a vapour and a gas with the formation of condensate is accompanied by two competing processes. On the one hand, mixing of the vapour with the gas leads to cooling, to the subsequent attainment of the dew point and the formation of liquid drops in the mixture. This mechanism, in particular, determines the coordinates and the law of motion of the left boundary of the intermediate zone $x^{(l)} = x^{(l)}(t)$. However, since the intermediate zone containing the condensate borders to the right on gas which is dry ($\rho_{ve} = 0$) at infinity, its right boundary, where evaporation occurs, has a finite coordinate $x^{(r)} = x^{(r)}(t)$. The rate of advance of this boundary is determined by the intensity of evaporation which, in its turn, depends on the diffusion transport of the vapour.

The limiting values of the initial temperatures T_o and T_e, at which the formation of condensate is observed, is determined from the condition that the self-similar coordinates of the boundaries are identical $\xi^{(l)} = \xi^{(r)}$. The corresponding curve (Fig. 3) divides the (T_o, T_e) coordinate plane into two domains. When the values of the initial temperatures T_o and T_e increase, there is a transition from one domain into the other across this curve which implies that there is a qualitative change in the mode of mixing, and condensate is not formed in the mixing zone.

3. Mixing of vapour–gas–condensate systems with a vapour-gas mixture

We will now consider a situation when, in the initial state in the left semi-infinite zone ($-\infty < x \le 0$), there is a homogeneous vapour–gas–condensate mixture at a temperature T_o with a partial density of the condensate ρ_{lo} and, in the right zone ($0 < x < \infty$), there is vapour-gas mixture with a partial density of the gas ρ_{ge} at a temperature T_e. The points of the upper boundary of the hatched area in Fig. 1 when $T < T_b$ correspond to the initial condition for the left zone. Then, according to expressions (1.4) and (1.5) for the partial vapour and gas densities ρ_{vo} and ρ_{go}, we have

$$\rho_{vo} = \frac{\mu_v}{RT_v} P_s(T_0), \quad \rho_{go} = \frac{\mu_g}{RT_o}(P - P_s(T_o)) \tag{3.1}$$

Generally speaking, all the points of the hatched area correspond to the initial state of the mixture in the right zone. In the case of the partial vapour density ρ_{ve} for states of a vapour-gas mixture, when the point representing this state lies inside the hatched area in Fig. 1, we have

$$\rho_{ve} = \frac{\mu_v}{RT_e} P - \frac{\mu_v}{\mu_g} \rho_{ge} \tag{3.2}$$

In the case when the vapour-gas mixture is located in the right zone, although $\rho_{le} = 0$ corresponds to the dew point, the expression for the partial vapour density ρ_{ve} will be analogous to expression (3.1) for ρ_{vo} and, in fact,

$$\rho_{ve} = \frac{\mu_v}{RT_e} P_s(T_e) \tag{3.3}$$

In this case, the initial conditions for Eqs. (1.1) and (1.2) can be written in the form

$$\begin{aligned} &T = T_o, \quad \rho_g = \rho_{go}, \quad \rho_l = \rho_{lo} \ \text{when} \ x < 0 \ (\xi \to -\infty) \\ &T = T_o, \quad \rho_g = \rho_{ge}, \quad \rho_l = 0 \ \text{when} \ x > 0 \ (\xi \to \infty) \end{aligned} \tag{3.4}$$

When $t > 0$, there is no partition, and mixing occurs with the formation of a frontal boundary with the coordinate $x = x^{(f)}$ ($\xi = \xi^{(f)}$) on which the partial density of the condensate undergoes a jump from a value of $\rho_l = \rho_l^{(f)}$ when $x = x^{(f)-}$ ($\xi = \xi^{(f)-}$) to $\rho_l = 0$ when $x = x^{(f)+}$ ($\xi = \xi^{(f)+}$). On this boundary, we can write conditions which are analogous to (2.2), (2.3) and (2.6) and, also, (2.13) and (2.14). The solution of Eq. (2.9), which satisfies conditions (3.4), is then written in the whole mixing zone in the form

$$\rho_g(\xi) = \rho_{go} + \frac{\rho_{ge} - \rho_{go}}{\sqrt{\pi}} E(-\infty, \xi) \tag{3.5}$$

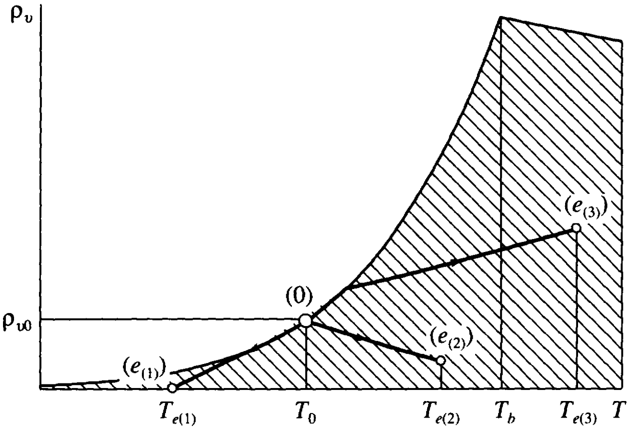

Fig. 4.

The solution for the temperature distribution in the zone $x < x^{(f)}$ $(\xi < \xi^{(f)})$ has a form similar to (2.16) when corresponding account is taken of expression (3.5). In the left zone $x < x^{(f)}$ $(\xi < \xi^{(f)})$, on the basis of the heat flux equation (2.10) and taking account of the initial condition for ρ_l $(\rho_l = \rho_{lo})$, we obtain

$$\rho_l = \rho_{lo} + \frac{1}{2}\int_{-\infty}^{\xi}\frac{\rho c}{l}\frac{d^2T}{d\rho_g^2}\left(\frac{d\rho_g}{d\xi}\right)^2\frac{d\xi}{\xi}; \quad \frac{d\rho_g}{d\xi} = \frac{\rho_{ge}-\rho_{go}}{\sqrt{\pi}}\exp(-\xi^2) \tag{3.6}$$

For the temperature distribution when $x > x^{(f)}$ $(\xi > \xi^{(f)})$, we have

$$T = T_e + (T^{(f)} - T_e)\frac{E(\xi, \infty)}{R(\xi^{(f)}, \infty)} \tag{3.7}$$

Two unknown parameters $\xi^{(f)}$ and $T^{(f)}$ appear in this solution. When account is taken of solutions (3.5) and (3.6), it is possible to obtain an equation which is analogous to (2.23):

$$\left(\frac{dT}{d\rho_g}\right)_{T^{(f)}}\frac{\rho_{go}-\rho_{ge}}{\sqrt{\pi}} + \frac{T^{(f)} - T_e}{E(\xi^{(f)}, \infty)} = 2\xi^{(f)}\frac{\rho_l^{(f)}l}{\rho c}\exp((\xi^{(f)})^2) \tag{3.8}$$

where

$$\left(\frac{dT}{d\rho_g}\right)_{T^{(f)}} = -\frac{RT^{(f)2}}{\mu_g}\left(P - P_s(T^{(f)})\left(1 - \frac{T_*}{T^{(f)}}\right)\right)^{-1}$$

On the other hand, from solution (3.6) when $\xi = \xi^{(f)}$, we have

$$\rho_l^{(f)} = \rho_{lo} + \frac{1}{2}\int_{-\infty}^{\xi^{(f)}}\frac{\rho c}{l}\frac{d^2T}{d\rho_g^2}\left(\frac{d\rho_g}{d\xi}\right)^2\frac{d\xi}{\xi} \tag{3.9}$$

Equations (3.8) and (3.9) enable us to determine the unknown self-similar coordinate of the frontal boundary of the phase transition $\xi^{(f)}$ and the value of the partial condensate density $\rho_l^{(f)}$ as viewed from the vapour–gas–condensate zone.

As an example, we will consider the mixing of moist water vapour which is a mixture of air, water vapour and droplets. In the calculations, the values of the parameters defining the initial state were chosen to be the conventional parameters[1] for a natural mist: an atmospheric pressure $P = 10^5$ Pa, a partial density of the droplets in the mixture $\rho_l = 10\,\mathrm{g/m^3}$, and a vapour temperature $T_o = 300$ K. The values of the two parameters T_e and $\rho_{\upsilon e}$ determine the character of the mixing. The initial value of the air temperature T_e was varied from 273 K to 400 K and the partial vapour density in the air $\rho_{\upsilon e}$ was varied from 0 up to a value corresponding to the dew point.

The trajectories for different versions of the mixing of a gas–vapour–condensate mixture with a vapour-gas mixture are shown in the phase diagram presented in Fig. 4. The initial state of the vapour–gas–condensate mixture is the same in all the versions and is determined by the initial temperature T_o and the initial partial densities $\rho_{\upsilon o}$, ρ_{go} and ρ_{lo}. Here, the values of $\rho_{\upsilon o}$ and ρ_{go} are related to the temperature T_o by formulae (3.1). The initial point (O) of the trajectory, which lies on the saturation curve, corresponds to this state. The existence of droplets of the liquid in the mixture, which is in an equilibrium state with the vapour, is only possible at a temperature which is lower than the boiling point T_b. Therefore, the points in the diagram corresponding to the initial state of the vapour–gas–condensate mixture are always located to the left of the boiling point $(T < T_b)$. The initial temperature T_e of the vapour-gas mixture in the right domain $(x > 0)$ can be located in three different intervals which are determined by the values of T_o and T_b. The partial vapour density $\rho_{\upsilon e}$ at a fixed temperature T_e can take values between zero and the equilibrium value (3.3) corresponding to the temperature T_e. The terminal point of the phase trajectory corresponds to the initial state of the vapour-gas mixture where there is no condensate.

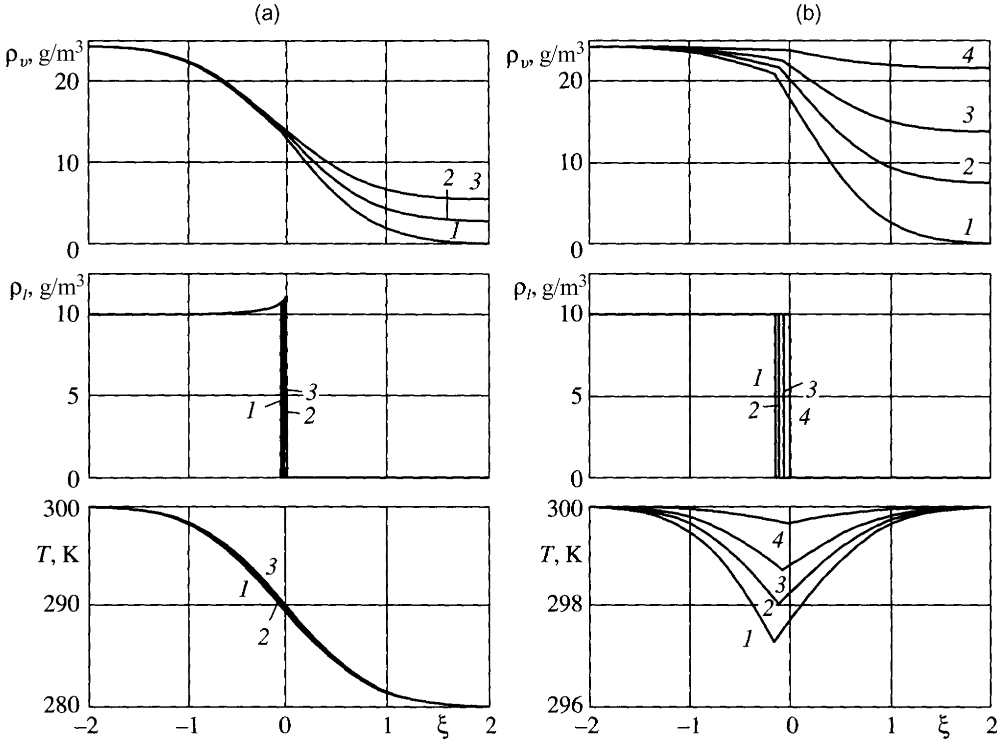

Fig. 5.

Mixing of a vapour–gas–condensate mixture with a gas which contains vapour and the subsequent changes in the temperature and partial vapour density can cause condensation in the domain $-\infty < x < x^{(f)}$. Moreover, evaporation of the droplets occurs on the frontal boundary ($x = x^{(f)}$). The intensity of evaporation is determined by two quantities: the difference in the initial temperatures $\Delta T = T_e - T_o$, as the intensity of the supply of heat to the frontal boundary ($x = x^{(f)}$) depends on this, and the difference in the initial values of the partial pressure of the vapour $\Delta\rho_v = \rho_{ve} - \rho_{vo}$, as the intensity of the removal of the vapour formed during evaporation from the boundary of the domains depends on the magnitude of this. Depending on the values of these parameters, evaporation or condensation can predominate in the zone to the left of the frontal boundary ($-\infty < x \le x^{(f)}$).

In accordance to what has been noted above, three characteristic versions of the mixing of a vapour–gas–condensate mixture with a vapour-gas mixture are considered in which the temperature values $T_e = T_{e(1)}, T_{e(2)}, T_{e(3)}$ are found in the three above-mentioned intervals (Fig. 4).

In the first version ($T_{e(1)} \le T_o$), mixing of a vapour–gas–condensate mixture with a colder vapour-gas mixture occurs (Fig. 5, *a*, the mixing of fog with cold gas: $1 - \rho_{ve} = 0$, $2 - \rho_{ve} = 2.9\,\mathrm{g/m^3}$, $3 - \rho_{ve} = 5.5\,\mathrm{g/m^3}$). In spite of the negative temperature drop ($\Delta T = T_e - T_o < 0$), evaporation occurs on the frontal boundary ($x = x^{(f)}$). This is due to the fact that the initial value of the partial vapour density ρ_{ve} in the domain of the vapour-gas mixture is always less than in the domain of the vapour–gas–condensate mixture ($\Delta\rho_v = \rho_{ve} - \rho_{vo} < 0$), and intense removal of the vapour which is formed from the frontal boundary towards the cold but "dryer" vapour-gas mixture occurs. A case of particular interest is when the initial values of the temperatures T_o and T_e are close or equal. Evaporation then leads to noticeable cooling in the mixing zone and to the formation of a temperature "well" $\Delta T^{(f)} = T_o - T^{(f)}$ (Fig. 5, *b*, the mixing of fog with a warm gas: $1 - \rho_{ve} = 0$, $2 - \rho_{ve} = 7.6\,\mathrm{g/m^3}$, $3 - \rho_{ve} = 13.9\,\mathrm{g/m^3}$, $4 - \rho_{ve} = 21.8\,\mathrm{g/m^3}$). The depth $\Delta T^{(f)}$ of this "well" depends on the intensity of evaporation and, consequently, it is determined by the difference in the initial values of the partial vapour density $\Delta\rho_v = \rho_{ve} - \rho_{vo}$ (when $\rho_{ve} = 0$, the cooling of the mixture on the frontal boundary is a maximum).

In the second version ($T_o < T_{e(2)} < T_b$), a vapour–gas–condensate mixture is mixed with a warmer vapour-gas mixture with a temperature below the boiling point of the liquid T_b (Fig. 6, the mixing of fog with a hot gas: $1 - \rho_{ve} = 0$, $2 - \rho_{ve} = 74.2\,\mathrm{g/m^3}$, $3 - \rho_{ve} = 123.7\,\mathrm{g/m^3}$, $4 - \rho_{ve} = 197.8\,\mathrm{g/m^3}$). An increase in the partial vapour density in the vapour-gas mixture ρ_{ve} leads to a reduction in the rate of evaporation on the frontal boundary, although it takes place because of the temperature drop ($T_{e(2)} > T^{(f)}$) even when the value in the vapour–gas–condensate mixture ($\rho_{ve} > \rho_{vo}$) above it is exceeded. However, in this case according to Fick's law, the vapour entering from the right zone, as well as that which has been formed due to evaporation on the frontal boundary ($x = x^{(f)}$), diffuses into the zone of the vapour–gas–condensate mixture, that is, in a direction which is the reverse of the usual direction. Therefore, in the zone of the vapour–gas–condensate mixture ($x \le x^{(f)}$), partial condensation of the vapour entering from the hotter but moister vapour-gas mixture ($x > x^{(f)}$) occurs, that is, intense absorption of the vapour by the vapour–gas–condensate mixture is observed.

As ρ_{ve} increases the intensity of the condensation becomes greater and it can even completely compensate for the evaporation. At the same time, the value of the partial condensate density $\rho_l^{(f)}$ on the boundary $x = x^{(f)}$ may exceed the initial value of ρ_{lo} several fold.

The third version ($T_{e(3)} \ge T_b$) corresponds to mixing of the vapour–gas–condensate mixture with a vapour-gas mixture with a temperature above the boiling point of the liquid in the droplets. As in the preceding version, the rate of evaporation becomes greater as the initial temperature of the vapour-gas mixture T_e increases but decreases when the initial partial vapour density ρ_{ve} is increased. However, here, the difference in the initial temperatures of the mixtures $\Delta T = T_e - T_o$ is so large that evaporation on the frontal boundary is possible even if the vapour–gas–condensate mixture is mixed with the pure superheated vapour ($\rho_{ge} = 0$).

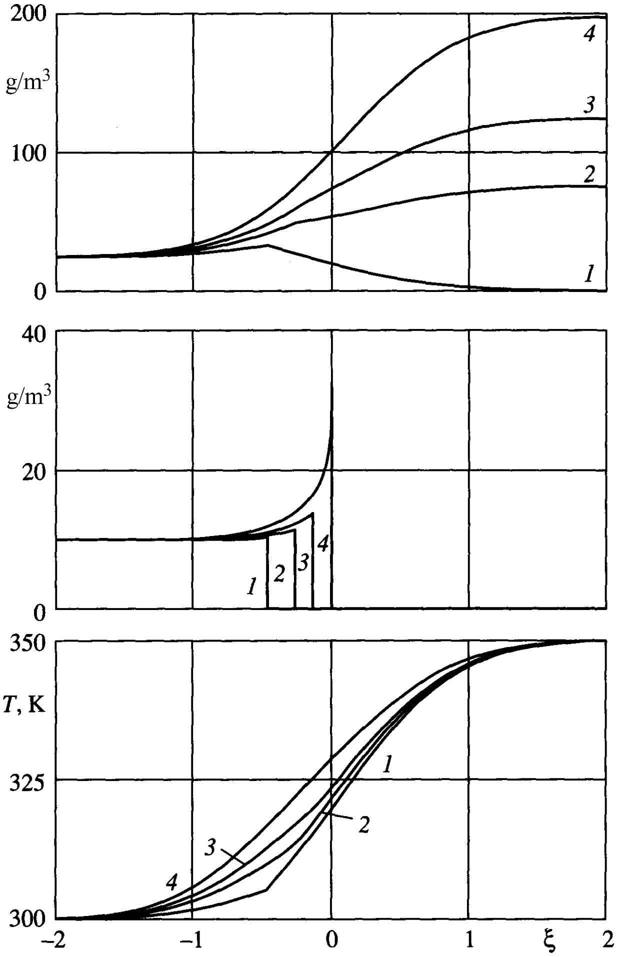

Fig. 6.

Hence, a chart of the different modes of mixing has been constructed from the solutions of one-dimensional problems of the mixing of vapour–gas–condensate mixtures which have been obtained. Initial values of the temperatures have been found for which the mixing of a vapour-gas mixture with a pure gas is accompanied by the formation of an intermediate layer with condensate.

The following regularities have been established by an analysis of the solutions describing the mixing of a vapour–gas–condensate mixture with a vapour-gas mixture.:

1) when the values of the initial temperatures of the vapour–gas–condensate mixture T_0 and the vapour-gas mixture T_e are close, cooling of the mixture to below the initial temperatures occurs in the mixing zone (a temperature "well" with a depth of several degrees is formed);
2) when the temperature T_e is higher than the value of the temperature T_0 and the partial vapour density in the vapour-gas mixture is higher than in the vapour–gas–condensate mixture ($\rho_{ve} > \rho_{v0}$), a significant growth in the condensate is observed, due to the vapour which is introduced into the domain of the vapour–gas–condensate mixture from the vapour-gas mixture;
3) when the temperature T_e is higher the the boiling point T_b, evaporation on the frontal boundary occurs for all values of the partial density ρ_{ve} (even during mixing with pure vapour).

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